The Verification on the Proof of Goldbach's Conjecture

Introduction

Goldbach's conjecture is one of the most famous unsolved problems in number theory. In 1742, Christian Goldbach sent a letter to Leonhard Euler. In it, he argued that $\forall\, n > 2,\ 2n = p + q,$ where $n \in \mathbb{N}$ and p, q are prime numbers. Many mathematicians and young researchers have spent much time attempting to solve this problem. Unfortunately, for 297 years ago nobody has proven this Conjecture. The first research on Goldbach's Conjecture was conducted in the 1920's. Goldbach's Conjecture is categorized into 2 types of statement, namely Strong Goldbach's Conjecture and Weak Goldbach's Conjecture. The Weak Goldbach's Conjecture, known as ternary Goldbach problem, states that $n = p + q + r,$ where $n \in \mathbb{N}$ and p, q, r are prime numbers. This is called a weak assumption because if the strong Goldbach's Conjecture which focuses on even number is proven, then the weak Goldbach's conjecture will true.

Some mathematicians have controversially claimed that they can prove Goldbach's Conjecture on Goldbach's Conjecture while others argued that it is still remain unproven . For instance Mathis (n.d) in his website has written an article about his proof of strong Goldbach Conjecture. He claims that he can prove Goldbach's Conjecture using probability theory. Mathi's proof is not valid because it is impossible to verify all infinite prime numbers which satisfy the Goldbach's Conjecture. We need to find a new theory as a method to prove this conjecture because the previous theory is not strong enough as a fundamental theory to solve this problem. I believe that Goldbach's Conjecture still remains unproven and further study is needed to prove it. This essay will examine some claims on the Goldbach's Conjecture and examines the truth of Goldbach's conjecture by providing some examples.

The claims of the Proof of Goldbach's Conjecture

Most of the common studies which are conducted in mathematics are concerned with proving the theorems. Mathematicians attempt to invent a proof as authors and the readers need to verify that proof. Journal editors or mathematics teachers are also keen to examine proofs of theorems (Andrzej Pelc, 2014). There is no doubt that Goldbach's Conjecture has caught the attention of some mathematicians to prove this conjecture. Schnirelman (1930, in Markakis, Provatidis, Markakis, 2013) shows that the Goldbach's Conjecture is true at most 20 primes and this result enriched by other authors such as Ramare, Jing-run Chen, Montgomery and Vaughan who showed larger prime number than their previous number (Markakis, Provatidis, Markakis, 2013). In fact, this study is not enough to use as a proof of Goldbach's Conjecture because there is a huge size of prime numbers that we must verify.

Another approach that are applied to solve this problem is probability theory. Mathis (n.d) in his website stated that Goldbach' Conjecture is an easy and simple problem. As a result, to prove this conjecture, he claimed that the simplest theory such as probability could be used. He calculated the probability of the pairs of prime and non prime numbers and transforms it into fraction. Consequently, his proof is not use probability theory but fraction. However, his proof is lack of definition and axioms and in my view, he still use probability theory although he change the probability of the pairs prime number into fractions.

In 1937, Vinogradov believed that for some constant C, there are all odd numbers less than C that holds Goldbach's Conjecture. The first constant C is 10^{1346} found by Liu-Wang, which cannot be checked by computer, then Hardy attempted to find less number that can be checked by computer. He found that computer could check even numbers up to 4^{18}, using this study, Hardy claimed that the largest even number in Goldbach's Conjecture which can be checked by computer is 10^{27}. He deduced that the weak Goldbach Conjecture is now proven. To prove a theorem or conjecture, it is not enough for only some specifics samples, the proof should hold for all even/odd numbers in mathematics. To support his research, Hardy also uses Fourier analysis as a method to prove the Conjecture. This might provide an estimate number in which hold Goldbach Conjecture (Hardy, in Oliveira e Silva, Herzog, & Pardi 2014). In fact, his study is not appropriate because there are infinite even numbers in mathematics that would be difficult to estimate with those theories.

Some researchers are keen on solving the Goldbach's problem by using Betrand's Postulate and Sieve method. They modified Bertrand's postulate which relate to Goldbach's Conjecture and show that $\forall\, n \geq 9, \exists p$ such that $\frac{n+1}{2} \leq p \leq n-4,\ then\ n-p\ is\ even\ \wedge n-p > 2$, therefore *n-p* is not prime, so their modification failed. Many studies done, none could prove the conjecture. Gerstein (1993) stated that Goldbach Conjecture has connection with twin prime conjecture($\forall\, primes\ p\ \ni p+21\ \in p$). This theory is not well-known, and he attempted to link both problems. He showed the equivalent statement with Goldbach's conjecture as follows:

$$\forall\, n \in N, n \geq 2, \exists\, k, p, q\ \in N, with\ 0 \leq k\ \leq n-2\ and\ with\ p, q\ prime, \exists\ n^2 - k^2 = pq\ (*)$$

It should use an opposite direction to prove Goldbach Conjecture. While it might not prove that (*) is true, but for k=1, this following conjecture is made:

$$\forall\, n \geq 2, \exists\, p, q\ primes\ \ni\ n^2 - 1 = pq\ (**)$$

It is immediately clear that (**) is equivalent to twin prime conjecture. Although both Goldbach and twin prime conjecture are not similar, they are equivalent. This research could be used as a fundamental theory to prove Goldbach Conjecture, because both Goldbach and twin prime conjecture have a similar pattern.

To prove the conjecture, Ruiz (2014) constructed two arithmetic sequences, namely sequence A and sequence B, with necessary conditions that for all natural number can be expressed as the sum of prime numbers. It is stated that a particular pair primes are always created between all non prime numbers in sequence A and sequence B. This is essential to construct "a non-probabilistic formula" which finds the approximation of prime numbers that satisfies the Goldbach Conjecture. The result allows verifying that Goldbach Conjecture is true. To complete the investigation, some axioms in arithmetic progression might be used. Ruiz (2014) created a formula as follows:

$$k(jx) \approx 1 - \frac{30\pi(bx)}{x - 2.5\pi(ax)}$$

to prove Goldbach's Conjecture. This formula is valid although the numerical results are slightly different from the analytical method. The proof which is claimed among mathematicians could not be checked.

Verification of Goldbach's Conjecture

Many theories both in number theory and analysis have been applied in order to prove Goldbach's Conjecture. Some mathematicians might assume that the Goldbach's Conjecture is true, but more studies are needed to verify that conjecture. To support this statement, it will show some researches who focus on verifying the truth of Goldbach's Conjecture. The Weak Goldbach's Conjecture or ternary conjecture is easily to prove than the Strong one, it makes some mathematicians prefer to verify the Weak one.

One of study that discusses the verification of Goldbach's Conjecture is Richestein's paper (2014). He explained two methods that are used to verify the binary Goldbach's Conjecture. Before we verify the number, it is needed to find the sets of Prime number P_1 and P_2 in the interval *[a,b]* such that:

$$\{2n \mid a \le 2n \le b\} \subseteq P_1 + P_2 = \{p_1 + p_2 \mid p_2 \in P_2, p_2 \in P_2\}$$

The first method shows:

$$P_1 = \{p \mid p \in [1, b - a + \delta]\}, \quad P_2 = \{p \mid p \in [a - \delta, a]\}$$

While, the second method shows:

$$P_1 = \{p \mid p \in [1, \delta]\}, \quad P_2 = \{p \mid p \in [a - \delta, b]\}$$

The second method is chosen by Richestein (2001) to apply into some machines, although the second method is slower than the first method In order to calculated using the second method, it used two PC's under Linux and was written in C language. The results show that it took around 130 days to generate the set P_2 between 10^{12} and 4.10^{14}. E Silva and Pardi (Helfgott, 2013) also verified some even integer between 4 and 4.10^{18} which can be represented and the sum of two primes. Verifying the integer using computers might waste time and money. An advanced computer is needed to calculate it. One of recent verification is conducted by Helfgott and Platt (2014) at 8.875 x 10^{30} using C programming language. Although use modern computer, it still took long time to calculate the numbers.

Conclusion

The proof that is claimed among mathematicians such as Mathis, Vinogradov, Gerstein , Hardy and Ruiz, although apply many theories, like, arithmetic progression, twin prime and sieve method is lack of definitions or axioms. Ruiz (2014) also showed the new formula to support his claim in the proof of Goldbach's Conjecture. It is a simple formula which only estimates the pair of prime number and poor evidence. The validity of theorem's proof should be checked in mathematical community. Unfortunately, the claims of Goldbach's conjecture are not acceptable and rejected by other mathematicians. The theory and the formula which is created by them are not valid and not true.

In my opinion, the Goldbach's Conjecture is unproven until now, because it is only an approach not a proof. They solve this problem only use the previous theories, such as Sieve, twin prime number, arithmetic progression and Ramanujan theorems, it cannot allow researcher to prove this complex conjecture and the old unsolved problem. Thus, new rules, law, and theory must be created to carry out this problem. I believe that it is possible to solve Goldbach's Conjecture using new method or just evolving the previous theorems or theory as a new approach to prove this conjecture and also the verification of Goldbach Conjecture using numerical computation shows that this conjecture is true.

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